Southwest Fisheries Center Administrative Report H-79-1

USING MARKOV DECISION MODELS AND RELATED TECHNIQUES FOR PURPOSES OTHER THAN SIMPLE OPTIMIZATION II: THE NORTHERN ANCHOVY, *ENGRAULIS MORDAX*

Roy Mendelssohn[1]

January 1979

[1]Southwest Fisheries Center, National Marine Fisheries Service, NOAA, Honolulu, HI 96812.

I. INTRODUCTION

In part I (Mendelssohn 1978b), optimization techniques for stochastic models were used to analyze policy choices for salmon runs in Bristol Bay, Alaska. In that instance, stochastic models of the fishery were available, but no comparable policy analysis. Policies that optimized total expected (discounted) yield were found; policies that were optimal when year to year fluctuations had a cost were also found; and estimates of the risk entailed due to uncertainty about estimates of the model's parameters were also considered.

In this paper, similar techniques are used to improve upon an existing analysis, that of the northern anchovy, *Engraulis mordax* (MacCall 1978; Pacific Fishery Management Council 1978). Policies are found that improve upon present policies on almost every management criteria, whether the main objective be biological (total harvest), economic (total expected discounted value of the catch), protection of the stock, or being certain that the industry is not shut down often and needlessly. Further, it is shown that the policy that maximizes the total harvest does not close the fishery 2 out of 3 years, as stated in the Pacific anchovy management plan (Pacific Fishery Management Council 1978). Finally, the optimal policies can be explained in terms of the underlying assumptions of the model used. This allows the decisionmaker to decide how much weight to give the results of the model, depending on how realistic he or she feels these critical assumptions are.

Two points should be emphasized at the outset. The present policies are based on a thoughtful look at past experience, and an insightful look at what were thought to be sound alternative policies. The problem arises in that these policies are selected on an ad hoc basis, and our intuition on stochastic models is not always as we would believe. The present paper improves upon this analysis by optimizing, for any given objective, over all possible decision rules. It is this fact that makes possible the finding of policies that are improvements on every criteria.

Secondly, the results presented here show how to improve upon the policy analysis for anchovy, but do not necessarily imply that the present management schemes, all things being considered, are not more acceptable. This seeming contradiction arises because a model's main purpose is insight; the actual decision process includes many factors not included in the model. These factors include balancing political pressures from the different interest groups, biological and economic intuition or insights gain from experience with the fishery, and the similarity with past management policies. The latter point is particularly important, for it is often difficult to implement radically different management schemes, and decisionmakers often feel "safer" (less risky) with "the devil they know," rather than the devil they don't know.

II. THE MODEL

The model used was developed by MacCall (1978) and is also described in detail in the Pacific anchovy management plan (Pacific Fishery Management Council 1978). Let x_t be the biomass at the start of period t, F_t the fishing effort during period t, and d_t a random variable that is independent and identically distributed through time. Then in general form the model is:

$$x_{t+1} = s[x_t, F_t, d_t]$$

where s[•, •, •] is given by:

$$s[x_t, F_t, d_t] = x_t\, e^{-(F_t + 0.8)} + \left(e^{d_t} \left(\frac{1}{3.649} + \left(\frac{1}{x_t} - \frac{1}{3.649} \right) 0.695 \right)^{-1} - x_t\, e^{-(0.8)} \right) e^{-0.152\, F_t} \qquad (2.1)$$

where d_t is distributed as N(0, 0.2294). This can be simplified somewhat by noting that:

$$\left(\frac{1}{3.649} + \left(\frac{1}{x_t} - \frac{1}{3.649} \right) 0.695 \right)^{-1} = \frac{3.649\, x_t}{\frac{0.695}{3.649} + 0.305\, x_t}$$

The transition has a deterministic term for mortality in the present biomass, and a stochastic term for larval recruitment to the fishery.

Catch, therefore, is a random variable also, given by:

$$C_t = \frac{F_t}{F_t + 0.8}\, x_t \left(1 - e^{-(F_t + 0.8)} \right) + 0.647 \left(e^{d} \left(\frac{3.649\, x_t}{0.190 + 0.305\, x_t} \right) - x_t\, e^{-0.8} \right) \times \frac{0.76\, F_t}{0.76\, F_t + 0.8} \left(1 - e^{-(0.152\, F_t + 0.16)} \right) \qquad (2.2)$$

To set this up in a manageable form for numerical work, four things need to be decided:

(i) an objective function,

(ii) the decision variable, and any constraints on its value,

(iii) a grid over which to discretize the problem,

(iv) a methodology to redefine the problem over the discrete grid.

For (i), two objective functions are used throughout. The first maximizes the expected total discounted harvest. More formally, if E is the expectation operator, the objective is to:

$$\text{maximize } E \sum_{t=1}^{\infty} \alpha^{t-1} C_t \qquad 0 \leq \alpha < 1 \tag{2.3}$$

where C_t is the random variable catch, and the system satisfies all the constraints in (2.1) and (2.2). If $\alpha = 1$, the equivalent criterion is to maximize the expected per period harvest, or

$$\text{maximize } \lim_{T \to \infty} E\left(\frac{\sum_{t=1}^{T} C_t}{T}\right)$$

Economic considerations are taken into account more explicitly, by using a one-period return of:

$$V(C_t, x_t) = (59.67 \times C_t) - (1.8953 \times 10^{-5} \times C_t^2) - (10315 \times C_t \times x_t^{-0.4}) \tag{2.4}$$

so that the objective is to maximize the expected total discounted economic value

$$\text{maximize } E \sum_{t=1}^{\infty} \alpha^{t-1} V(C_t, x_t) \tag{2.5}$$

The derivation of $V(C_t, x_t)$ is given in full detail in the Pacific anchovy management plan (Pacific Fishery Management Council 1978). However, it bears closer examination. For fixed x_t, $V(\cdot, x_t)$ is concave in C_t. Therefore, taking the partial derivative with respect to C_t:

$$V^{[1]}(C_t, x_t) = 59.67 - (3.7906 \times 10^{-5} \times C_t) - (10{,}315 \times x_t^{-0.4})$$

It can readily be seen that for $x \leq 392{,}900.8661$ this function achieves its maximum at zero, where it has a value of zero. This states, however indirectly, that the industry prefers being shut down if the population size is too small, and that it has no fixed costs when shut down. As will be seen, many of the optimal policies exploit the fact that the industry faces zero costs when not harvesting.

For each run, the stationary distribution of the Markov chain induced by an optimal policy is calculated. This makes it possible to calculate the average catch per period, its variance and standard deviation, the median population size and catch, and the percent time the population is less than a given amount. The latter is particularly important in that recreational fishers have come out in favor of no commercial fishing if the population falls below 1 million tons. Thus the percentage of time there is no fishing can be calculated. It will be seen that this "cutoff," per se, does not by itself accomplish what appears to be the true goal of the recreationalists, that is to keep the probability that the population is less than 1 million tons at an acceptable level.

For (ii), there are two possible decision variable, catch or effort. In the Pacific anchovy management plan catch is used (Pacific Fishery Management Council 1978). Here, effort is the decision variable. The reason for this is that catch is a random variable. Thus, any observed catch could have come from an almost infinite combination of values of F_t and d_t. However, the value of F_t is needed to calculate the transition probabilities. It is not clear in MacCall (1978) and the Pacific anchovy management plan (Pacific Fishery Management Council 1978), how the iterative procedure described finds the value of F_t. On the other hand, F_t is a deterministic variable in the model; and, once given an optimal policy, it is easy to calculate the <u>expected catch</u> in a period for any combination of x_t and F_t.

A more important problem is what bounds should be put on F_t? Clearly, $F_t \geq 0$. However, at the present time the fishers do not have the capacity for unlimited effort. The analysis of an expected catch quota will be very different if it is assumed that the entire quota is taken compared with an analysis where it is assumed that only part of the quota is taken. However, information does not exist to determine which of these is the actual case. To look at this problem, runs were done with successively increasing upper bounds on F_t. These runs were analyzed for two things: where the policy no longer changed as the bound increased; and where the expected catch from most population sizes became much larger than anything ever caught before. Upper bounds of 0.1, 0.2, 0.4, and 0.6 were tried. It will be seen that a constraint of the form $0 \leq F_t \leq 0.1$ or $0 \leq F_t \leq 0.2$ seems to be the most realistic.

Also, to examine the effects of limiting harvests if the population size is less than 1 million tons, identical runs were done where $F_t \equiv 0$ if $x_t \leq 1$ million tons.

A grid size of 50 equally spaced points between 0.073 and 3.649 was used. Previous computational experience (Mendelssohn 1978a) suggests that a 50-100 point grid is necessary to insure numerical accuracy, particularly when calculating the tail probabilities of the stationary distribution. Using 3.649 as an upper bound (actually all states $\geq$ 3.649 are put into this state) is a conservative procedure, in that it rules out infrequent but very large "potlatches" that would occur otherwise. This makes it less desirable to allow the population to get very large; lowers the estimate of the mean catch; and overestimates the probability of being in the lower population sizes. Thus, if the policies are an improvement in each of these categories, we can feel safe that the grid choice, if anything, made it more difficult to obtain improvements.

Equation (2.1) was discretized as follows. Let Φ be the standard normal integral. From (2.1):

$$\Pr\left\{x_{t+1} \leq \omega\right\} = \Pr\left\{s[x_t,\ F_t,\ d_t] \leq \omega\right\} = \Pr\left\{d \leq \ln\left(a(x_t)\right)\right\}$$

where $a(x_t)$ is given by equation (2.1).

Then from the normality of d:

$$\Pr\left\{d \leq \ln(a)\right\} = \Phi\left(\frac{\ln(a)}{\sigma}\right)$$

where $\sigma = 0.479$. Let x_1, x_2 be two adjacent points on the grid. For any F_t,

$$\Pr\left\{x_{t+1} \leq x_2 \mid x_t, F_t\right\} = \Phi\left(\frac{\ln\ a(x_2)}{\sigma}\right) - \Phi\left(\frac{\ln\ a(x_1)}{\sigma}\right)$$

that is, the total probability of going to any state in the interval $(x_1, x_2]$. The mathematics and convergence issues of this procedure are discussed in Bertsekas (1975) and Whitt (1978); note that this procedure is different from that used in MacCall (1978) and the Pacific anchovy management plan (Pacific Fishery Management Council 1978), where point densities are used, and then are normalized by dividing through by the sum of the probabilities.

The point estimate of expected catch is used. (It would be a worthwhile effort to repeat this analysis trying differing returns--for example, averaging the returns aggregated into any state-action pair, as in Mendelssohn 1978a.) This is calculated by noting that the random variable enters into (2.2) at only one spot:

$$0.647\ (e^d)\ \frac{3.649\, x_t}{0.190 + 0.305\ x_t}$$

It is well known that if $d \sim N(m, \sigma^2)$, then e^d is a lognormal random variable with mean $\exp\left\{m + \tfrac{1}{2}\sigma^2\right\}$. Therefore, the expected catch is:

$$\frac{F_t}{F_t+0.8}x_t\left(1-e^{-(F_t+0.8)}\right) + 0.647\left(1.1216\left(\frac{3.649\,x_t}{0.190+0.305\,x_t}\right) - x_t C^{-(0.8)}\right)$$

$$\times\ \frac{0.76\,F_t}{0.76\,F_t+0.8}\left(1-e^{-(0.152\,F_t+0.16)}\right)$$

All runs are solved using dynamic programming, using several techniques to accelerate convergence and eliminate actions (Porteus 1971; Hastings and van Nunen 1977). It is well known that if the one-period return is a random variable, then the solution to (2.3) or (2.4) is the limiting solution to the following system of recursive equations:

$$f_o(\cdot) \equiv 0 \tag{2.6}$$

$$f_n(x) = \max_{0 \le F \le F_{max}} \left\{ G(x,\ F) + \alpha\, E\, f_{n-1}\ (s[x,\ F,\ d]) \right\}$$

where $G(x,\ F)$ is the expected one-period return.

III. RESULTS

A total of 17 runs were performed, each with a discount factor of $\alpha = 0.97$. This is in the range of discount factors that would arise if it is assumed that the true interest rate is the present interest rate less the rate of inflation. Test runs were performed for discount factors ranging from 0.95 to 0.99, with little change in the results. The results are summarized in Tables 1(a-p). In the latter tables, for each state, optimal effort is given, the

Table 1

expected one-period catch is given, and the cumulative percent of time (in the long run) the population is less than or equal to the given state is also given. At the end, summary statistics of each run are given.

As mentioned before, it is difficult to compare these runs with present options in terms of mean catch per period, and its variance, because if larger effort is allowed, these figures will increase. Instead, we will see that policies have been found that produce expected catches as large as if not much larger than anything the commercial fishery has caught to date, at the same time producing population statistics that improve upon the present policies in every category.

The first question is, which of the runs is the correct model? In the 6-year period 1970-76, the commercial fishery had catches of 96,242; 44,853; 69,100; 132,636; 82,817, and 158,511 tons. A look at Tables 1(c and d) show that this is well within the range of catches if $F_{max} = 0.1$. In fact, at that maximum fishing effort, the mean catch is roughly 176,500 tons and the median catch is 174,000 tons, assuming the United States gets it all. However, Mexico is building up its reduction fishery, and cooperative management is the goal. This may mean the United States would be allotted only 50%-70% of the catch for a given year. If the United States would get only 70% of the catch, this is still a reasonable model, because roughly 40% of the time the expected yearly catch would be greater than any previous United States catch.

If the United States retains only 50% of the catch, then $F_{max} = 0.1$ would be restrictive on growth in either the United States or the growing Mexican fishery. However, if $F_{max} = 0.2$, then even if the United States were to get only 50% of the expected catch each year, Tables 1(e-h) predict roughly 50% of the time the expected catch to the United States would be higher than any previous catch, and 25% of the time the expected United States catch would be 50% greater than any previous catch.

Thus, in terms of actually modeling the present fishery $F_{max} = 0.1$ or 0.2 appears to be the most realistic assumption. The other runs show what might be expected if there is unlimited growth in fishing capacity, be it from the United States or from Mexico.

Let $F_{max} = 0.2$. Two runs were performed with no other restrictions on the fishery, one with expected total discounted harvest as the objected, and the other with expected total discounted economic value as the objective. These are summarized in Tables 1 (e and g). As mentioned earlier, both optimal policies, even assuming the United States gets only 50% of the expected catch each year, produce catches well in excess of the largest amount yet caught by the United States commercial fishery.

When expected total discounted harvest is the criterion, an optimal policy has no catch only 1.07% of the time, and the population size is less than 0.5 million tons 3.3% of the time, and less than

1 million tons only 17.13% of the time. These compare more than favorably with any policy given in the Pacific anchovy management plan (Pacific Fishery Management Council 1978) (see Table 2 for the comparable statistics). When expected total discounted economic value is the criterion, there is no fishery only 8.8% of the time, and the population is less than 0.5 million tons only 1.97% of the time, and less than 1.0 million tons only 14.72% of the time. Again these statistics improve upon every comparable statistic in the Pacific anchovy management plan, while allowing for expansion in the total fishery.

Table 2

Comparable statistics are given in Tables 1 (b-d) assuming that $F_{max} = 0.1$. Again, each optimal policy improves upon any of the comparable policies in the Pacific anchovy management plan. The question then is what is gained by not allowing harvesting for population sizes less than 1 million tons or less than 0.5 million tons. The latter is examined in Table 1 (d), and the only major difference is the increase in the percent years there is no fishery.

Tables 1 (f and h) give comparable runs to the two discussed previously, except now if the population size is less than 1 million tons, no harvesting is allowed. What is noticeable is that particularly when expected total discounted economic value is the criterion, little or no change occurs in the percent time the population is less than 1 million tons, but the percent time there is no fishery increases dramatically. To achieve a reduction in the percent time the fishery is less than 1 million tons requires either a cutoff point greater than a million tons and a concomitant increase in the number of years there is no fishery, or else a decrease in the fishing effort, which has the effect of limiting industry expansion.

The remaining runs show the probable effect of allowing a much larger commercial fishery. There are four noticeable trends when following an optimal policy.

Firstly, as the mean catch per period increase, so does the variance, the percent time there is no fishery, and the percent time the population is less than 1 million tons.

Secondly, an optimal policy for the expected total discounted value does not change as F_{max} increases above 0.4. This implies, that according to the model, unless economic conditions change, the industry will not want to expand beyond this point.

Thirdly, adding the "cutoff" at 1 million tons does not produce much of a change in the percent time the population is less than 1 million tons, while it always increases significantly the percent time there is no harvest.

Finally, for runs where the objective is to maximize expected total discounted economic return, no harvesting occurs at population sizes less than 876,000 tons, which is not very different than the avowed position of the recreationalists.

IV. DISCUSSION

More important than finding optimal policies is to discover why the particular policies found are optimal, and what this says about the strength and weaknesses of the model. The first consideration is when "amount harvested" is the only consideration, why is the fishery fished so hard? The second question is, why, even with economic considerations included, is it optimal not to fish with ever increasing frequency as maximum fishing effort increases? And finally, why doesn't a cutoff produce the desired result of protecting the stock?

The first question, and to some extent the last question, can be answered by examining more closely equation (2.1). Note again, there is a deterministic part representing mortality of present biomass, and a random part representing recruitment <u>which only depends on biomass at the beginning of the period</u>. Even with no fishing, in 1 year 55% of the present biomass dies, and in 2 years 80% of the standing biomass dies. Since recruitment, according to the model, only depends on biomass levels at the beginning of the period, the "discounted" value (discounted by mortality) of a ton of biomass is quite low. If it isn't harvested now, it will pretty much be dead anyway, and will add little additional to the recruitment 2 years hence. Thus, it is desirable to harvest hard. Cutoffs do not protect the stock, because the recruitment next year, which is a large part of the population, only depends on the population size before harvesting.

The answer to the second question has already been alluded to in section II. There it is shown that the one-period economic return function is such that at certain biomasses the return will be negative if harvesting commences, <u>but zero if there is no harvesting</u>. Clearly, however, there is a cost to the industry from not harvesting; in fact, there is probably a cost if the catch is below a certain minimum level; all of which would make it more desirable from the industry's point of view, to harvest a small amount rather than none. When the model does not specify such "smoothing costs" (or fixed costs, or start-up and shutdown costs) an optimal policy will not be concerned with how frequently no fishing occurs. And this is reflected in the policies found here, as well as the present policies discussed in the Pacific anchovy management plan (Pacific Fishery Management Council 1978).

Finally, we have already answered partially the third question. But more importantly, the question remains is a "cutoff" the underlying aim of the recreationalist? Probably not. More than likely, what they desire is to keep the probability that the fishery is less than 1 million tons at acceptable levels, whether it be by "cutoff," reduction of fishing effort, or whatever.

All models represent restrictions and abstractions of reality, and these shortcomings should be guides to use and not reason for dismissal. Given the results in section III, and the present capacity of fishery, good policies can be found that allow for expansion in the fishery, almost never have no fishing, and have a smaller chance for low population sizes than present policies. However, if the fishery expands to a point where maximum fishing effort is greater than 0.2, it will be important to do two things. First, to more properly evaluate the impact on industry, fixed costs and "smoothing costs" should be researched, to include the real costs faced by industry if there is no harvest.

From the recreationalist point of view, the stochastic dynamic program (2.6) can be optimized subject to a constraint on the long-run probability of having a standing biomass less than 1 million tons. This constraint can parametrically be made tighter and tighter, so that the tradeoff between this constraint and the value of the fishery can be examined. Details can be found in Mendelssohn (1977).

Finally, all the policies considered depend on knowing the standing biomass each year. This can be a costly undertaking. How much would be lost if the estimate were obtain only every 2 years, or 3 years, as compared to the savings from not obtaining the estimate? How should an optimal policy be modified to take into consideration this decrease in information about (or increase in risk in) the system? These questions will be examined in a future paper.

ACKNOWLEDGMENTS

I am grateful for Alec MacCall for allowing me to see an early version of his paper (MacCall 1978), invaluable computer assistance from Debra Chow, and David Stoutemyer provided several helpful discussions on improving the computer program used.

LITERATURE CITED

Bertsekas, D. P.

1975. Dynamic programming and stochastic control. Academic Press, chapter 5:179-221.

Hastings, N. A. J., and J. H. E. E. van Nunen.

1977. The action elimination algorithm for Markovian decision processes. In H. C. Tijms and J. Wessels (editors), Markov decision theory, p. 161-170. Mathematical Center Tract No. 93, Amsterdam.

MacCall, A.

1978. Population models for the northern anchovy (Engraulis mordax). Paper presented at Symposium on the Biological Basis of Pelagic Fish Stock (ICES).

Mendelssohn, R.

1977. Determining the best tradeoff between expected economic return and the risk of undesirable events when managing a randomly varying population. Southwest Fisheries Center Admin. Rep. 13H, 1977. Natl. Mar. Fish. Serv., NOAA, Honolulu, Hawaii, 24 p.

1978a. The effects of grid size and approximation techniques on the solutions of Markov decision problems. Southwest Fisheries Center Admin. Rep. 20H, 1978. Natl. Mar. Fish. Serv., NOAA, Honolulu, Hawaii, 14 p.

Mendelssohn, R.

1978b. Using Markov decision models and related techniques for purposes other than simple optimization I: Analyzing the the consequences of policy alternatives on the management of salmon runs. Southwest Fisheries Center Admin. Rep. 27H, 1978. Natl. Mar. Fish. Serv., NOAA, Honolulu, Hawaii, 24 p.

Pacific Fishery Management Council.

1978. Environmental impact statement and fishery management plan for the northern anchovy fishery. Pacific Fishery Management Council, Portland, Oregon.

Porteus, E.

1971. Some bounds for discounted sequential decision processes. Manage. Sci. 18:7-11.

Whitt, W.

1978. Approximations to dynamic programs I. Math. Oper. Res. 3:231-243.

Table 1 (a).--Base case--no harvesting allowed.

State	Stationary distribution
0.073	0.0000
0.146	0.0001
0.219	0.0005
0.292	0.0014
0.365	0.0031
0.438	0.0057
0.511	0.0094
0.584	0.0143
0.657	0.0205
0.730	0.0280
0.803	0.0368
0.876	0.0468
0.949	0.0580
1.022	0.0740
1.095	0.0839
1.168	0.0984
1.241	0.1138
1.314	0.1301
1.387	0.1472
1.460	0.1649
1.533	0.1832
1.606	0.2020
1.679	0.2213
1.752	0.2409
1.824	0.2608
1.897	0.2808
1.970	0.3010
2.043	0.3212
2.116	0.3414
2.189	0.3615
2.262	0.3814

Table 1 (a).--Continued.

State	Stationary distribution
2.335	0.4012
2.408	0.4207
2.481	0.4400
2.554	0.4590
2.627	0.4776
2.700	0.4959
2.773	0.5138
2.846	0.5313
2.919	0.5484
2.992	0.5651
3.065	0.5813
3.138	0.5971
3.211	0.6124
3.284	0.6273
3.357	0.6417
3.430	0.6557
3.503	0.6692
3.576	0.6823
$\geq$3.649	1.0000

Median population = 2.700

% time population is $\leq$ 0.5 = 0.57%

% time population is $\leq$ 1.0 = 5.8%

Table 1 (b).--Total discounted harvest, effort restricted to be no greater than 0.1.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0000
0.146	0.100	0.011	0.0004
0.219	0.100	0.017	0.0018
0.292	0.100	0.023	0.0046
0.365	0.100	0.028	0.0092
0.438	0.100	0.034	0.0156
0.511	0.100	0.039	0.0239
0.584	0.100	0.045	0.0340
0.657	0.100	0.051	0.0457
0.730	0.100	0.056	0.0590
0.803	0.100	0.062	0.0737
0.876	0.100	0.067	0.0897
0.949	0.100	0.073	0.1069
1.022	0.100	0.078	0.1251
1.095	0.100	0.084	0.1441
1.168	0.100	0.089	0.1638
1.241	0.100	0.094	0.1841
1.314	0.100	0.100	0.2049
1.387	0.100	0.105	0.2261
1.460	0.100	0.111	0.2476
1.533	0.100	0.116	0.2692
1.606	0.100	0.121	0.2909
1.679	0.100	0.127	0.3126
1.752	0.100	0.132	0.3342
1.824	0.100	0.137	0.3557
1.897	0.100	0.143	0.3770
1.970	0.100	0.148	0.3980
2.043	0.100	0.153	0.4187
2.116	0.100	0.158	0.4391
2.189	0.100	0.164	0.4591

Table 1 (b).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.262	0.100	0.169	0.4787
2.335	0.100	0.174	0.4979
2.408	0.100	0.179	0.5167
2.481	0.100	0.185	0.5350
2.554	0.100	0.190	0.5528
2.627	0.100	0.195	0.5701
2.700	0.100	0.200	0.5869
2.773	0.100	0.205	0.6032
2.846	0.100	0.210	0.6190
2.919	0.100	0.216	0.6343
2.992	0.100	0.221	0.6491
3.065	0.100	0.226	0.6634
3.138	0.100	0.231	0.6772
3.211	0.100	0.236	0.6905
3.284	0.100	0.241	0.7033
3.357	0.100	0.246	0.7157
3.430	0.100	0.251	0.7276
3.503	0.100	0.256	0.7391
3.576	0.100	0.262	0.7501
$\geq$3.649	0.100	0.267	1.0000

Mean catch = 0.1765

Variance = 0.00555

Standard deviation = 0.07448

% time no catch = 0.0%

Median population size = 2.335

% time population is $\leq$ 0.5 = 1.56%

% time population is $\leq$ 1.0 = 10.69%

Expected catch is roughly 0.074 x population if population is greater than 0.073.

Table 1 (c).--Total economic return, effort restricted to be no greater than 0.1.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0000
0.146	0.000	0.000	0.0002
0.219	0.000	0.000	0.0011
0.292	0.000	0.000	0.0032
0.365	0.000	0.000	0.0071
0.438	0.000	0.000	0.0129
0.511	0.000	0.000	0.0207
0.584	0.000	0.000	0.0304
0.657	0.044	0.023	0.0419
0.730	0.100	0.056	0.0551
0.803	0.100	0.062	0.0698
0.876	0.100	0.067	0.0858
0.949	0.100	0.073	0.1030
1.022	0.100	0.078	0.1212
1.095	0.100	0.084	0.1403
1.168	0.100	0.089	0.1601
1.241	0.100	0.094	0.1806
1.314	0.100	0.100	0.2015
1.387	0.100	0.105	0.2228
1.460	0.100	0.111	0.2444
1.533	0.100	0.116	0.2661
1.606	0.100	0.122	0.2879
1.679	0.100	0.127	0.3097
1.752	0.100	0.132	0.3314
1.824	0.100	0.137	0.3530
1.897	0.100	0.143	0.3744
1.970	0.100	0.148	0.3955
2.043	0.100	0.153	0.4163
2.116	0.100	0.158	0.4368
2.189	0.100	0.164	0.4569

Table 1 (c).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.262	0.100	0.169	0.4766
2.335	0.100	0.174	0.4959
2.408	0.100	0.179	0.5147
2.481	0.100	0.185	0.5331
2.554	0.100	0.190	0.5510
2.627	0.100	0.195	0.5684
2.700	0.100	0.200	0.5853
2.773	0.100	0.205	0.6017
2.846	0.100	0.210	0.6176
2.919	0.100	0.216	0.6330
2.992	0.100	0.221	0.6479
3.065	0.100	0.226	0.6623
3.138	0.100	0.231	0.6762
3.211	0.100	0.236	0.6896
3.284	0.100	0.241	0.7025
3.357	0.100	0.246	0.7149
3.430	0.100	0.251	0.7269
3.503	0.100	0.256	0.7384
3.576	0.100	0.262	0.7495
$\geq$3.649	0.100	0.267	1.0000

Mean catch = 0.1756

Variance = 0.00586

Standard deviation = 0.07658

% time no catch = 3.04%

Median population size = 2.335

% time population is $\leq$ 0.5 = 1.29%

% time population is $\leq$ 1.0 = 10.30%

Table 1 (d).--Total discounted harvest and total economic return, effort restricted to be no more than 0.1, no harvesting if population size is less than 1 million tons.

Mean harvest = 0.1729

Variance = 0.00691

Standard deviation = 0.08310

% time no harvest = 9.59%

Median population size = 2.335

% time population is $\leq$ 0.5 = 1.05%

% time population is $\leq$ 1.0 = 9.59%

Table 1 (e).--Total discounted harvest, effort restricted to be no greater than 0.2.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0000
0.146	0.000	0.000	0.0009
0.219	0.000	0.000	0.0042
0.292	0.000	0.000	0.0107
0.365	0.200	0.055	0.0204
0.438	0.200	0.065	0.0330
0.511	0.200	0.076	0.0480
0.584	0.200	0.087	0.0651
0.657	0.200	0.098	0.0840
0.730	0.200	0.108	0.1043
0.803	0.200	0.119	0.1258
0.876	0.200	0.129	0.1482
0.949	0.200	0.140	0.1713
1.022	0.200	0.150	0.1949
1.095	0.200	0.161	0.2189
1.168	0.200	0.171	0.2430
1.241	0.200	0.182	0.2672
0.314	0.200	0.192	0.2914
1.387	0.200	0.203	0.3154
1.460	0.200	0.213	0.3392
1.533	0.200	0.223	0.3627
1.606	0.200	0.234	0.3858
1.679	0.200	0.244	0.4085
1.752	0.200	0.254	0.4308
1.824	0.200	0.264	0.4526
1.897	0.200	0.274	0.4738
1.970	0.200	0.285	0.4945
2.043	0.200	0.295	0.5146
2.116	0.200	0.305	0.5342

Table 1 (e).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.200	0.315	0.5532
2.262	0.200	0.325	0.5716
2.335	0.200	0.335	0.5894
2.408	0.200	0.345	0.6066
2.481	0.200	0.355	0.6232
2.554	0.200	0.365	0.6392
2.627	0.200	0.375	0.6546
2.700	0.200	0.385	0.6695
2.773	0.200	0.395	0.6838
2.846	0.200	0.405	0.6976
2.919	0.200	0.415	0.7108
2.992	0.200	0.425	0.7235
3.065	0.200	0.435	0.7357
3.138	0.200	0.444	0.7474
3.211	0.200	0.454	0.7586
3.284	0.200	0.464	0.7693
3.357	0.200	0.474	0.7796
3.430	0.200	0.484	0.7895
3.503	0.200	0.494	0.7989
3.576	0.200	0.503	0.8079
$\geq$3.649	0.200	0.513	1.0000

Mean catch = 0.3055

Variance = 0.02217

Standard deviation = 0.14890

% time no catch = 1.07%

Median population size = 1.970

% time population is $\leq$ 0.5 = 3.3%

% time population is $\leq$ 1.0 = 17.13%

Table 1 (f).--Total discounted harvest, effort restricted to be no greater than 0.2, no harvest if population size is less than 1 million tons.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0000
0.146	0.000	0.000	0.0002
0.219	0.000	0.000	0.0014
0.292	0.000	0.000	0.0043
0.365	0.000	0.000	0.0095
0.438	0.000	0.000	0.0175
0.511	0.000	0.000	0.0283
0.584	0.000	0.000	0.0418
0.657	0.000	0.000	0.0577
0.730	0.000	0.000	0.0758
0.803	0.000	0.000	0.0957
0.876	0.000	0.000	0.1171
0.949	0.000	0.000	0.1396
1.022	0.200	0.150	0.1630
1.095	0.200	0.161	0.1871
1.168	0.200	0.171	0.2116
1.241	0.200	0.182	0.2363
1.314	0.200	0.192	0.2611
1.387	0.200	0.203	0.2859
1.460	0.200	0.213	0.3105
1.533	0.200	0.223	0.3349
1.606	0.200	0.234	0.3589
1.679	0.200	0.244	0.3825
1.752	0.200	0.254	0.4057
1.824	0.200	0.264	0.4284
1.897	0.200	0.274	0.4506
1.970	0.200	0.285	0.4722
2.043	0.200	0.295	0.4932

Table 1 (f).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.116	0.200	0.305	0.5136
2.189	0.200	0.315	0.5334
2.262	0.200	0.325	0.5526
2.335	0.200	0.335	0.5712
2.408	0.200	0.345	0.5891
2.481	0.200	0.355	0.6064
2.554	0.200	0.365	0.6231
2.627	0.200	0.375	0.6392
2.700	0.200	0.385	0.6547
2.773	0.200	0.395	0.6696
2.846	0.200	0.405	0.6840
2.919	0.200	0.415	0.6978
2.992	0.200	0.425	0.7111
3.065	0.200	0.434	0.7238
3.138	0.200	0.444	0.7360
3.211	0.200	0.454	0.7477
3.284	0.200	0.464	0.7589
3.357	0.200	0.474	0.7697
3.430	0.200	0.484	0.7800
3.503	0.200	0.494	0.7899
3.576	0.200	0.503	0.7993
$\geq$3.649	0.200	0.513	1.0000

Mean catch = 0.3008

Variance = 0.02812

Standard deviation = 0.16769

% time no harvest = 13.96%

Median population size = 2.043

% time population is $\leq$ 0.5 = 1.75%

% time population is $\leq$ 1.0 = 13.96%

Table 1 (g).--Total discounted economic value, effort restricted to be no greater than 0.2.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0000
0.146	0.000	0.000	0.0003
0.219	0.000	0.000	0.0016
0.292	0.000	0.000	0.0049
0.365	0.000	0.000	0.0108
0.438	0.000	0.000	0.0197
0.511	0.000	0.000	0.0315
0.584	0.000	0.000	0.0460
0.657	0.000	0.000	0.0629
0.730	0.000	0.000	0.0818
0.803	0.048	0.050	0.1024
0.876	0.124	0.082	0.1243
0.949	0.188	0.132	0.1472
1.022	0.200	0.150	0.1709
1.095	0.200	0.161	0.1951
1.168	0.200	0.171	0.2197
1.241	0.200	0.182	0.2444
1.314	0.200	0.192	0.2692
1.387	0.200	0.203	0.2939
1.460	0.200	0.213	0.3184
1.533	0.200	0.223	0.3426
1.606	0.200	0.234	0.3664
1.679	0.200	0.244	0.3898
1.752	0.200	0.254	0.4127
1.824	0.200	0.264	0.4351
1.897	0.200	0.274	0.4570
1.970	0.200	0.285	0.4783
2.043	0.200	0.295	0.4991
2.116	0.200	0.305	0.5193
2.189	0.200	0.315	0.5389

Table 1 (g).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.262	0.200	0.325	0.5579
2.335	0.200	0.335	0.5762
2.408	0.200	0.345	0.5939
2.481	0.200	0.355	0.6110
2.554	0.200	0.365	0.6275
2.627	0.200	0.375	0.6434
2.700	0.200	0.385	0.6587
2.773	0.200	0.395	0.6735
2.846	0.200	0.405	0.6877
2.919	0.200	0.415	0.7013
2.992	0.200	0.425	0.7144
3.065	0.200	0.435	0.7270
3.138	0.200	0.444	0.7391
3.211	0.200	0.454	0.7507
3.284	0.200	0.464	0.7618
3.357	0.200	0.474	0.7724
3.430	0.200	0.484	0.7826
3.503	0.200	0.494	0.7923
3.576	0.200	0.503	0.8016
$\geq$3.649	0.200	0.513	1.0000

Mean catch = 0.3031

Variance = 0.02593

Standard deviation = 0.16103

% time no catch = 8.18%

Median population size = 2.043

% time population is $\leq$ 0.5 = 1.97%

% time population is $\leq$ 1.0 = 14.72%

Table 1 (h).--Total discounted economic value, effort restricted to be greater than 0.2, no harvest if population is less than 1 million tons

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0000
0.146	0.000	0.000	0.0002
0.219	0.000	0.000	0.0014
0.292	0.000	0.000	0.0043
0.365	0.000	0.000	0.0095
0.438	0.000	0.000	0.0175
0.511	0.000	0.000	0.0283
0.584	0.000	0.000	0.0418
0.657	0.000	0.000	0.0577
0.730	0.000	0.000	0.0758
0.803	0.000	0.000	0.0957
0.876	0.000	0.000	0.1171
0.949	0.000	0.000	0.1396
1.022	0.200	0.150	0.1630
1.095	0.200	0.161	0.1871
1.168	0.200	0.171	0.2116
1.241	0.200	0.182	0.2363
1.314	0.200	0.192	0.2611
1.387	0.200	0.203	0.2859
1.460	0.200	0.213	0.3105
1.533	0.200	0.223	0.3349
1.606	0.200	0.234	0.3589
1.679	0.200	0.244	0.3825
1.752	0.200	0.254	0.4057
1.824	0.200	0.264	0.4284
1.897	0.200	0.274	0.4506
1.970	0.200	0.285	0.4722
2.043	0.200	0.295	0.4932

Table 1 (h).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.116	0.200	0.305	0.5136
2.189	0.200	0.315	0.5334
2.262	0.200	0.325	0.5526
2.335	0.200	0.335	0.5712
2.408	0.200	0.345	0.5891
2.481	0.200	0.355	0.6064
2.554	0.200	0.365	0.6231
2.627	0.200	0.375	0.6392
2.700	0.200	0.385	0.6547
2.773	0.200	0.395	0.6696
2.846	0.200	0.405	0.6840
2.919	0.200	0.415	0.6978
2.992	0.200	0.425	0.7111
3.065	0.200	0.435	0.7238
3.138	0.200	0.444	0.7360
3.211	0.200	0.454	0.7477
3.284	0.200	0.464	0.7589
3.357	0.200	0.474	0.7697
3.430	0.200	0.484	0.7800
3.503	0.200	0.494	9.7899
3.576	0.200	0.503	0.7993
$\geq$3.649	0.200	0.513	1.0000

Mean catch = 0.3008

Variance = 0.02812

Standard deviation = 0.16769

% time no catch = 13.96%

Median population size = 2.043

% time population is $\leq$ 0.5 = 1.75%

% time population is $\leq$ 1.0 = 13.96%

Table 1 (i).--Total discounted harvest, effort restricted to be no greater than 0.4.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0001
0.146	0.000	0.000	0.0017
0.219	0.000	0.000	0.0072
0.292	0.000	0.000	0.0183
0.365	0.000	0.000	0.0353
0.438	0.000	0.000	0.0576
0.511	0.000	0.000	0.0840
0.584	0.000	0.000	0.1134
0.657	0.056	0.029	0.1447
0.730	0.320	0.166	0.1771
0.803	0.400	0.221	0.2100
0.876	0.400	0.241	0.2428
0.949	0.400	0.260	0.2752
1.022	0.400	0.280	0.3070
1.095	0.400	0.299	0.3380
1.168	0.400	0.319	0.3680
1.241	0.400	0.338	0.3970
1.314	0.400	0.357	0.4250
1.387	0.400	0.377	0.4519
1.460	0.400	0.396	0.4778
1.533	0.400	0.415	0.5026
1.606	0.400	0.434	0.5264
1.679	0.400	0.453	0.5491
1.752	0.400	0.472	0.5708
1.824	0.400	0.491	0.5916
1.897	0.400	0.510	0.6114
1.970	0.400	0.528	0.6303
2.043	0.400	0.547	0.6484
2.116	0.400	0.567	0.6656

Table 1 (i).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.400	0.585	0.6820
2.262	0.400	0.604	0.6976
2.335	0.400	0.623	0.7125
2.408	0.400	0.641	0.7267
2.481	0.400	0.660	0.7402
2.554	0.400	0.678	0.7530
2.627	0.400	0.697	0.7652
2.700	0.400	0.715	0.7768
2.773	0.400	0.734	0.7878
2.846	0.400	0.752	0.7983
2.919	0.400	0.770	0.8083
2.992	0.400	0.789	0.8178
3.065	0.400	0.807	0.8268
3.138	0.400	0.825	0.8354
3.211	0.400	0.843	0.8435
3.284	0.400	0.861	0.8512
3.357	0.400	0.880	0.8585
3.430	0.400	0.898	0.8655
3.503	0.400	0.916	0.8721
3.576	0.400	0.934	0.8784
$\geq$3.649	0.400	0.952	1.0000

Mean catch = 0.4600

Variance = 0.08860

Standard deviation = 0.29766

% time no catch = 11.34%

Median population size = 1.533

% time population is $\leq$ 0.5 = 5.76%

% time population is $\leq$ 1.0 = 27.52%

Table 1 (j).--Total discounted harvest, effort restricted to be no greater than 0.4, no harvest allowed if population is less than 1 million tons.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0001
0.146	0.000	0.000	0.0013
0.219	0.000	0.000	0.0053
0.292	0.000	0.000	0.0134
0.365	0.000	0.000	0.0261
0.438	0.000	0.000	0.0434
0.511	0.000	0.000	0.0648
0.584	0.000	0.000	0.0897
0.657	0.000	0.000	0.1174
0.730	0.000	0.000	0.1471
0.803	0.000	0.000	0.1781
0.876	0.000	0.000	0.2099
0.949	0.000	0.000	0.2419
1.022	0.400	0.280	0.2738
1.095	0.400	0.300	0.3054
1.168	0.400	0.319	0.3363
1.241	0.400	0.338	0.3665
1.314	0.400	0.357	0.3958
1.387	0.400	0.377	0.4241
1.460	0.400	0.396	0.4514
1.533	0.400	0.415	0.4776
1.606	0.400	0.434	0.5028
1.679	0.400	0.453	0.5269
1.752	0.400	0.472	0.5500
1.824	0.400	0.491	0.5720
1.897	0.400	0.510	0.5930
1.970	0.400	0.529	0.6131
2.043	0.400	0.547	0.6322
2.116	0.400	0.567	0.6504

Table 1 (j).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.400	0.585	0.6678
2.262	0.400	0.604	0.6843
2.335	0.400	0.623	0.7000
2.408	0.400	0.641	0.7150
2.481	0.400	0.660	0.7292
2.554	0.400	0.678	0.7427
2.627	0.400	0.697	0.7555
2.700	0.400	0.715	0.7677
2.773	0.400	0.734	0.7793
2.846	0.400	0.752	0.7903
2.919	0.400	0.770	0.8008
2.992	0.400	0.789	0.8107
3.065	0.400	0.807	0.8201
3.138	0.400	0.825	0.8291
3.211	0.400	0.843	0.8376
3.284	0.400	0.861	0.8457
3.357	0.400	0.880	0.8534
3.430	0.400	0.898	0.8607
3.503	0.400	0.916	0.8676
3.576	0.400	0.934	0.8742
$\geq$3.649	0.400	0.952	1.0000

Mean catch = 0.4501

Variance = 0.10431

Standard deviation = 0.32297

% time no catch = 24.19%

Median population size = 1.606

% time population is $\leq$ 0.5 = 4.34%

% time population is $\leq$ 1.0 = 24.19%

Table 1 (k).--Total discounted economic value, effort restricted to be no greater than 0.4.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0001
0.146	0.000	0.000	0.0011
0.219	0.000	0.000	0.0046
0.292	0.000	0.000	0.0116
0.365	0.000	0.000	0.0227
0.438	0.000	0.000	0.0380
0.511	0.000	0.000	0.0572
0.584	0.000	0.000	0.0798
0.657	0.000	0.000	0.1051
0.730	0.000	0.000	0.1325
0.803	0.000	0.000	0.1614
0.876	0.056	0.038	0.1914
0.949	0.120	0.086	0.2218
1.022	0.176	0.134	0.2524
1.095	0.216	0.173	0.2828
1.168	0.256	0.215	0.3128
1.241	0.280	0.247	0.3423
1.314	0.304	0.281	0.3711
1.387	0.328	0.317	0.3991
1.460	0.344	0.347	0.4262
1.533	0.352	0.372	0.4524
1.606	0.360	0.396	0.4776
1.679	0.368	0.422	0.5019
1.752	0.376	0.448	0.5252
1.824	0.384	0.474	0.5476
1.897	0.384	0.492	0.5690
1.970	0.384	0.511	0.5895
2.043	0.392	0.538	0.6091
2.116	0.392	0.557	0.6278

Table 1 (k).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.392	0.575	0.6457
0.262	0.384	0.583	0.6628
2.335	0.384	0.601	0.6791
2.408	0.384	0.619	0.6946
2.481	0.384	0.637	0.7094
2.554	0.384	0.655	0.7235
2.627	0.376	0.661	0.7369
2.700	0.376	0.678	0.7497
2.773	0.368	0.683	0.7619
2.846	0.368	0.700	0.7735
2.919	0.368	0.717	0.7845
2.992	0.360	0.720	0.7950
3.065	0.360	0.737	0.8050
3.138	0.352	0.739	0.8145
3.211	0.352	0.755	0.8235
3.284	0.352	0.772	0.8321
3.357	0.344	0.772	0.8403
3.430	0.344	0.788	0.8481
3.503	0.336	0.788	0.8555
3.576	0.336	0.803	0.8625
$\geq$3.649	0.336	0.819	1.0000

Mean catch = 0.4184

Variance = 0.08364

Standard deviation = 0.28921

% time no catch = 16.14%

Median population size = 1.679

% time population is $\leq$ 0.5 = 3.8%

% time population is $\leq$ 1.0 = 22.18%

Table 1 (ℓ).--Total discounted economic value, effort restricted to be no greater than 0.4, no harvest if population less than 1 million tons.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0001
0.146	0.000	0.000	0.0011
0.219	0.000	0.000	0.0045
0.292	0.000	0.000	0.0113
0.365	0.000	0.000	0.0221
0.438	0.000	0.000	0.0370
0.511	0.000	0.000	0.0557
0.584	0.000	0.000	0.0777
0.657	0.000	0.000	0.1025
0.730	0.000	0.000	0.1295
0.803	0.000	0.000	0.1581
0.876	0.000	0.000	0.1877
0.949	0.000	0.000	0.2179
1.022	0.176	0.134	0.2484
1.095	0.216	0.172	0.2788
1.168	0.256	0.215	0.3088
1.241	0.280	0.247	0.3383
1.314	0.304	0.281	0.3671
1.387	0.328	0.317	0.3952
1.460	0.344	0.347	0.4224
1.533	0.352	0.372	0.4487
1.606	0.360	0.396	0.4741
1.679	0.368	0.422	0.4985
1.752	0.376	0.448	0.5220
1.824	0.384	0.474	0.5445
1.897	0.384	0.492	0.5661
1.970	0.384	0.511	0.5868
2.043	0.392	0.538	0.6066
2.116	0.392	0.557	0.6255

Table 1 (ℓ).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.392	0.575	0.6435
2.262	0.384	0.583	0.6607
2.335	0.384	0.601	0.6771
2.408	0.384	0.619	0.6927
2.481	0.384	0.637	0.7076
2.554	0.376	0.655	0.7218
2.627	0.376	0.661	0.7353
2.700	0.376	0.678	0.7482
2.773	0.368	0.683	0.7605
2.846	0.368	0.700	0.7722
2.919	0.368	0.717	0.7833
2.992	0.360	0.720	0.7939
3.065	0.360	0.737	0.8040
3.138	0.352	0.739	0.8136
3.211	0.352	0.755	0.8227
3.284	0.352	0.772	0.8314
3.357	0.344	0.772	0.8396
3.430	0.344	0.788	0.8474
3.503	0.336	0.788	0.8548
3.576	0.336	0.803	0.8619
$\geq$3.649	0.336	0.819	1.0000

Mean catch = 0.4168

Variance = 0.08600

Standard deviation = 0.29322

% time no catch = 21.79%

Median population size = 1.679

% time population is $\leq$ 0.5 = 3.7%

% time population is $\leq$ 1.0 = 21.79%

Table 1 (m).--Total discounted harvest, effort restricted to be no greater than 0.6.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0003
0.146	0.000	0.000	0.0023
0.219	0.000	0.000	0.0083
0.292	0.000	0.000	0.0202
0.365	0.000	0.000	0.0385
0.438	0.000	0.000	0.0630
0.511	0.000	0.000	0.0927
0.584	0.000	0.000	0.1265
0.657	0.000	0.000	0.1631
0.730	0.000	0.000	0.2013
0.803	0.000	0.000	0.2403
0.876	0.072	0.049	0.2792
0.949	0.264	0.180	0.3175
1.022	0.444	0.306	0.3548
1.095	0.600	0.419	0.3908
1.168	0.600	0.446	0.4253
1.241	0.600	0.473	0.4582
1.314	0.600	0.500	0.4895
1.387	0.600	0.527	0.5191
1.460	0.600	0.554	0.5471
1.533	0.600	0.581	0.5735
1.606	0.600	0.607	0.5984
1.679	0.600	0.634	0.6218
1.752	0.600	0.660	0.6438
1.824	0.600	0.687	0.6645
1.897	0.600	0.713	0.6839
1.970	0.600	0.740	0.7022
2.043	0.600	0.766	0.7194
2.116	0.600	0.792	0.7355

Table 1 (m).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.600	0.818	0.7506
2.262	0.600	0.844	0.7648
2.335	0.600	0.870	0.7782
2.408	0.600	0.896	0.7908
2.481	0.600	0.922	0.8026
2.554	0.600	0.948	0.8137
2.627	0.600	0.974	0.8242
2.700	0.600	0.999	0.8340
2.773	0.600	1.025	0.8433
2.846	0.600	1.050	0.8520
2.919	0.600	1.076	0.8602
2.992	0.600	1.102	0.8679
3.065	0.600	1.127	0.8752
3.138	0.600	1.152	0.8820
3.211	0.600	1.178	0.8884
3.284	0.600	1.203	0.8945
3.357	0.600	1.228	0.9002
3.430	0.600	1.253	0.9056
3.503	0.600	1.279	0.9107
3.576	0.600	1.304	0.9155
$\geq$3.649	0.600	1.329	1.0000

Mean catch = 0.5348

Variance = 0.18809

Standard deviation = 0.43369

% time no catch = 24.03%

Median population size = 1.387

% time population is ≤ 0.5 = 6.3%

% time population is ≤ 1.0 = 31.75%

For $x \geq 1.095$, expected catch is roughly $0.356x + 0.029$

Table 1 (n).--Total discounted harvest, effort restricted to be no greater than 0.6, no harvesting allowed if population is less than 1 million tons.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0003
0.146	0.000	0.000	0.0022
0.219	0.000	0.000	0.0080
0.292	0.000	0.000	0.0194
0.365	0.000	0.000	0.0369
0.438	0.000	0.000	0.0603
0.511	0.000	0.000	0.0888
0.584	0.000	0.000	0.1214
0.657	0.000	0.000	0.1569
0.730	0.000	0.000	0.1942
0.803	0.000	0.000	0.2324
0.876	0.000	0.000	0.2708
0.949	0.000	0.000	0.3088
1.022	0.444	0.306	0.3459
1.095	0.600	0.419	0.3819
1.168	0.600	0.446	0.4165
1.241	0.600	0.473	0.4496
1.314	0.600	0.500	0.4811
1.387	0.600	0.527	0.5110
1.460	0.600	0.554	0.5393
1.533	0.600	0.581	0.5660
1.606	0.600	0.607	0.5912
1.679	0.600	0.634	0.6150
1.752	0.600	0.660	0.6374
1.824	0.600	0.687	0.6584
1.897	0.600	0.713	0.6782
1.970	0.600	0.740	0.6968
2.043	0.600	0.766	0.7143
2.116	0.600	0.792	0.7307

Table 1 (n).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.600	0.818	0.7461
2.262	0.600	0.844	0.7606
2.335	0.600	0.870	0.7742
2.408	0.600	0.896	0.7870
2.481	0.600	0.922	0.7990
2.554	0.600	0.948	0.8103
2.627	0.600	0.974	0.8209
2.700	0.600	0.999	0.8309
2.773	0.600	1.025	0.8403
2.846	0.600	1.050	0.8492
2.919	0.600	1.076	0.8575
2.992	0.600	1.102	0.8654
3.065	0.600	1.127	0.8728
3.138	0.600	1.152	0.8798
3.211	0.600	1.178	0.8864
3.284	0.600	1.203	0.8926
3.357	0.600	1.228	0.8984
3.430	0.600	1.253	0.9039
3.503	0.600	1.279	0.9091
3.576	0.600	1.304	0.9140
$\geq$3.649	0.600	1.329	1.0000

Mean catch = 0.5337

Variance = 0.19546

Standard deviation = 0.442113

% time no catch = 30.88%

Median population size = 1.387

% time population is $\leq$ 0.5 = 6.03%

% time population is $\leq$ 1.0 = 30.88%

Table 1 (o).--Total discounted economic return, effort restricted to be no greater than 0.6.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0001
0.146	0.000	0.000	0.0011
0.219	0.000	0.000	0.0046
0.292	0.000	0.000	0.0116
0.365	0.000	0.000	0.0227
0.438	0.000	0.000	0.0380
0.511	0.000	0.000	0.0572
0.584	0.000	0.000	0.0797
0.657	0.000	0.000	0.1050
0.730	0.000	0.000	0.1324
0.803	0.000	0.000	0.1613
0.876	0.060	0.041	0.1912
0.949	0.120	0.086	0.2216
1.022	0.168	0.128	0.2521
1.095	0.216	0.173	0.2825
1.168	0.252	0.212	0.3125
1.241	0.288	0.253	0.3419
1.314	0.312	0.288	0.3706
1.387	0.324	0.314	0.3985
1.460	0.336	0.340	0.4256
1.533	0.360	0.368	0.4518
1.606	0.360	0.396	0.4770
1.679	0.372	0.426	0.5013
1.752	0.372	0.444	0.5246
1.824	0.384	0.474	0.5270
1.897	0.384	0.492	0.5484
1.970	0.384	0.511	0.5689
2.043	0.384	0.529	0.5885
2.116	0.384	0.547	0.6072

Table 1 (o).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.384	0.565	0.6251
2.262	0.384	0.583	0.6422
2.335	0.384	0.601	0.6585
2.408	0.384	0.619	0.6740
2.481	0.384	0.637	0.6888
2.554	0.384	0.655	0.7029
2.627	0.372	0.655	0.7164
2.700	0.372	0.672	0.7292
2.773	0.372	0.689	0.7414
2.846	0.372	0.706	0.7530
2.919	0.360	0.703	0.7641
2.992	0.360	0.720	0.7746
3.065	0.360	0.737	0.7846
3.138	0.360	0.754	0.7941
3.211	0.348	0.748	0.8032
3.284	0.348	0.764	0.8118
3.357	0.348	0.780	0.8200
3.430	0.348	0.796	0.8278
3.503	0.336	0.788	0.8352
3.576	0.336	0.803	0.8422
$\geq$3.649	0.336	0.819	1.0000

Mean catch = 0.4181

Variance = 0.08349

Standard deviation = 0.28895

% time no catch = 16.13%

Median population size = 1.679

% time population is $\leq$ 0.5 = 3.8%

% time population is $\leq$ 1.0 = 22.16%

Table 1 (p).--Total discounted economic value, effort restricted to be no greater than 0.6, no harvesting if population is less than 1 million tons.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
0.073	0.000	0.000	0.0001
0.146	0.000	0.000	0.0011
0.219	0.000	0.000	0.0045
0.292	0.000	0.000	0.0113
0.365	0.000	0.000	0.0221
0.438	0.000	0.000	0.0369
0.511	0.000	0.000	0.0555
0.584	0.000	0.000	0.0775
0.657	0.000	0.000	0.1023
0.730	0.000	0.000	0.1292
0.803	0.000	0.000	0.1577
0.876	0.000	0.000	0.1863
0.949	0.000	0.000	0.2165
1.022	0.168	0.128	0.2469
1.095	0.216	0.173	0.2772
1.168	0.252	0.212	0.3072
1.241	0.288	0.253	0.3367
1.314	0.312	0.288	0.3655
1.387	0.324	0.314	0.3936
1.460	0.336	0.340	0.4208
1.533	0.348	0.368	0.4471
1.606	0.360	0.396	0.4725
1.679	0.372	0.426	0.4969
1.752	0.372	0.444	0.5204
1.824	0.384	0.474	0.5429
1.897	0.384	0.492	0.5645
1.970	0.384	0.511	0.5852
2.043	0.384	0.529	0.6050
2.116	0.384	0.547	0.6239

Table 1 (p).--Continued.

State (tons x 10^6)	Optimal effort	Expected catch	Stationary distribution
2.189	0.384	0.565	0.6419
2.262	0.384	0.583	0.6591
2.335	0.384	0.601	0.6755
2.408	0.384	0.619	0.6912
2.481	0.384	0.637	0.7061
2.554	0.384	0.655	0.7203
2.627	0.372	0.655	0.7338
2.700	0.372	0.672	0.7467
2.773	0.372	0.689	0.7590
2.846	0.372	0.706	0.7707
2.919	0.360	0.703	0.7818
2.992	0.360	0.720	0.7924
3.065	0.360	0.737	0.8025
3.138	0.360	0.754	0.8121
3.211	0.348	0.748	0.8212
3.284	0.348	0.764	0.8299
3.357	0.348	0.780	0.8381
3.430	0.348	0.796	0.8459
3.503	0.336	0.788	0.8533
3.576	0.336	0.803	0.8604
$\geq$3.649	0.336	0.819	1.0000

Mean catch = 0.4164

Variance = 0.08601

Standard deviation = 0.29328

% time no catch = 21.65%

Median population size = 1.679

% time population is $\leq$ 0.5 = 3.69%

% time population is $\leq$ 1.0 = 21.65%

Table 2.--Statistics for present anchovy management alternatives (from Pacific Fishery Management Council 1978).

	Policy					
	1	2	3	4	5	6
% time population is ≤ 0.5	3.4%	4.4%	5.1%	3.0%	8.5%	9.6%
% time population is ≤ 1.0	16.1%	20.0%	20.8%	14.1%	30.3%	32.3%
% time no fishery	16.1%	20.0%	5.1%	14.1%	30.3%	9.6%